

# Quadrature formula, Orthogonal polynomials and its applications in solving linear Regular and Singular Integral Equations

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## OUTLINE OF THE TALK

In this note, we outline the following areas of numerical analysis:

- At the beginning we introduce space of functions such as norm space, inner product space and  $L_p$  space followed by the properties with some examples.
- Review simple quadrature formulas such as Midpoint rule, Trapezoidal rule, Simpson rule and its error estimates in different class of functions followed by product rule and its estimation.
- Orthogonal polynomials (mainly Chebyshev and Legendre) and its properties together with least-square approximation. Discrete orthogonally and Chebyshev interpolation followed by Gauss-Chebyshev quadrature formula and its error analysis.
- Review of integral equations of the first and second kind. Kernel approximation: Degenerate kernel approximation, interpolatory kernel approximation in integral equation. Nystrom method for continuous kernel functions.
- Lastly, application of the orthogonal polynomials in solving singular and hypersingular integral equations (HSIEs) of the first kind are demonstrated. Projection method together with Chebyshev polynomials of the first, second, third and fourth kinds are used to find bounded, unbounded and semi-bounded solutions of HSIEs respectively. Exact calculations of hypersingular and singular integrals for Chebyshev polynomials allow us to obtain high accurate approximate solution. Gauss-Chebyshev quadrature with Gauss-Lobatto nodes are presented as the high accurate computation of regular kernel integrals. Many examples are provided to verify the validity and accuracy of the proposed method. Numerical examples reveal that approximate solutions are exact if solution of HSIEs is of the polynomial forms with corresponding weights. It is worth to note that proposed method works well for large value of node points and errors are drastically decreases. SPU times are also shown to present effectiveness of the method and less complexity computations.

**Keywords:** Integral equations, Hypersingular integral equations, Chebyshev polynomials, Approximation, Convergence.